Comment on "Quantum Pump for Spin and Charge Transport in a Luttinger Liquid"

In their interesting paper [1], Sharma and Chamon address the issue of pumping in the context of Luttinger liquids. They claim that for spinless electrons (i) in the fast (UV) limit, the charge pumped per cycle is 0 (e) for repulsive, $g < 1$, (attractive, $g > 1$) interaction; (ii) this result is reversed in the adiabatic (IR) limit. Here we contest (i). We show that this limit is non-universal, and that for any given pumping frequency ω_0 and bare backscattering strength $\tilde{X}$ the pumped charge is *larger* for the *smaller* g. We complement our Comment by a proof of (ii).

Ref. [1] considers a pump made of two gates placed a distance $2a$ apart and biased with ac voltages of the same frequency ω_0. The pumping of spinless electrons is described [1] by the low-energy limit of the bosonized action

$$S = \int dt \Big\{ \int dx \frac{v}{2g} \left[(\partial_x \Phi)^2 + \frac{1}{v^2} (\partial_t \Phi)^2 \right] + X(t) \exp(i\sqrt{4\pi}\Phi(x=0)) + h.c. \Big\} \qquad (1)$$

where the charge density is $e\partial_x \Phi/\sqrt{\pi}$. The charge pumped per cycle is $q = e[\Phi(x=0, t=2\pi/\omega_0) - \Phi(x=0, t=0)]/\sqrt{\pi}$. $X(t)$ is an effective parameter that depends on the UV cutoff frequency $\Omega_c = v_F/a$ and the bare backscattering amplitudes $\tilde{X}_1, \tilde{X}_2$ at the two constrictions. The renormalization group (RG) analysis of Ref. [2] shows that at the same bare backscattering amplitudes the renormalized backscattering amplitude X is greater for repulsive than for attractive electrons.

To estimate the charge q pumped per cycle in the UV limit $\omega_0 \sim \Omega_c$ we apply second order perturbation theory in X [2]. We find that $q \sim e[X/(hv)]^2(\omega_0/\Omega_c)^{2g-2} \sim e[X/(hv)]^2$. This expression is valid as $X/(hv) \ll 1$. In this case $q = 0$. As shown below at $X/(hv) \gg 1$ the charge pumped per cycle is e. Thus, contrary to (i) the charge pumped per cycle is nonuniversal and can be either 0 or e (or any mean value between them) for a given g. Moreover, the relation between X and the bare backscattering amplitudes shows that for the same bare amplitudes the pumped charge is larger the smaller is g. This contradiction with (i) is due to the peculiar definition of the UV limit employed in Ref. [1]: $\omega_0 \gg \omega_\Gamma$ where ω_Γ is a crossover frequency depending on Ω_c and X. As X is small (large) at $g > 1$ ($g < 1$) the crossover frequency $\omega_\Gamma > \Omega_c$. Since $\omega_0 < \Omega_c$, the UV limit in the sense of Ref. [1] does not exist in the above regimes. Thus, the UV limit employed in Ref. [1] is just the limit of strong (weak) backscattering at $g > 1$ ($g < 1$). It is the backscattering amplitude and not the attractive or repulsive nature of the interaction that determines the pumped charge in the UV limit.

At the same time, (ii) is true and can be simply understood with the following argument. We rewrite the backscattering contribution to the action as $\Gamma(t)\cos(\sqrt{4\pi}\Phi(0,t) - \alpha(t))$ where $\Gamma(t) > 0$ is real. $\Gamma(t)$ and $\alpha(t)$ change at time intervals $\Delta t \sim 1/\omega_0$. At shorter time-scales the RG approach of Ref. [2] can be used since Γ and α may be considered as time-independent. One finds that Γ grows (decreases) under the action of the RG at $g < 1$ ($g > 1$), while the phase α remains unchanged. At $g > 1$ at the RG scale ω_0 we obtain a renormalized action with a small $\Gamma_R(t)$. The Keldysh perturbative expansion in $\Gamma_R(t)$ [3] shows that the charge pumped per cycle is much smaller than e. At $g < 1$ we obtain a large renormalized $\Gamma_R(t)$ at the scale ω_0. Hence, fluctuations about the minimum of the backscattering term in the action $(\sqrt{4\pi}\Phi(x=0) - \alpha) = -\pi/2$ are small. The charge pumped per cycle is $q = e\Delta\Phi(0)/\sqrt{\pi} = e\Delta\alpha/(2\pi) = en$, where $\Delta\alpha = 2\pi n$ is the change of the phase α per period.

In conclusion we confirm the result (ii) for the IR limit but show that the prediction (i) about the UV case as well as related predictions for electrons with spin are invalid and due to a peculiar definition of the UV limit used in Ref. [1].

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